Fluid Flow “Localization” in Disordered Systems

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Screening Effects

Laplacian System

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temperature

heat flux

Makarov theorem (1985): “The information dimension of the harmonic measure is equal to 1 in $d=2$.”

Meaning: The set where the activity takes place has a dimension equal to 1. The length of the active zone is proportional to the system size.

$$L_a \propto L^\alpha \quad \text{with} \quad \alpha = 1.$$
Makarov Theorem (for engineers!!)

- **substrate**
- **“alive” interface**
- **active interface**
Screening Effects in Flow through Rough Channels

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A surprising similarity is found between the distribution of hydrodynamic stress on the wall of an irregular channel and the distribution of flux from a purely Laplacian field on the same geometry. This finding is a direct outcome of numerical simulations of the Navier-Stokes equations for flow at low Reynolds numbers in two-dimensional channels with rough walls presenting either deterministic or random self-similar geometries. For high Reynolds numbers, the distribution of wall stresses on deterministic and random fractal rough channels becomes substantially dependent on the microscopic details of the walls geometry. Finally, the effects on the flow behavior of the channel symmetry and aspect ratio are also investigated.

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Screening in flow through fractal channels

Laplace & Stokes

\[ \phi_S \propto \phi_L \]

highly heterogeneous!!


Andrade, Araújo, Filoche & Sapoval, PRL (2007)
"Localization" in Flow through Disordered Porous Media

**Geometry**
- The porous medium is percolation-like: the disorder is binary, namely, sites are present on a square lattice with probability $p = 1 - \varepsilon$, where $\varepsilon$ is the porosity.

**Phenomenology**
- Newtonian fluid under continuum, isothermal, incompressible, and steady-state flow conditions.

**Continuity (mass conservation)**
\[ \nabla \cdot \mathbf{u} = 0 \]

**Navier-Stokes Equation (momentum conservation)**
\[ \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} \]

- Reynolds number - $\text{Re} \equiv \rho V d_p / \mu$
"Localization" in Flow through Disordered Porous Media

Stream Function Plots $\psi(x,y)$

incompressible 2d flows $\Rightarrow u \equiv \partial \psi / \partial y$ and $v \equiv -\partial \psi / \partial x$

stronger channeling!

Low Reynold (Stokes flow)  
High Reynolds (but laminar flow!)
"Localization" in Flow through Disordered Porous Media


Participation & Reynolds

\[ \pi \equiv \left( n \sum_{i=1}^{n} q_i^2 \right)^{-1} \]

\[ q_i \equiv e_i / \sum_{j=1}^{n} e_j \]

\[ e_i \propto (u_i^2 + v_i^2) \]

('localized' flow field) \[ \frac{1}{n} \leq \pi \leq 1 \] (equal partition of energy)
Non-Newtonian Flow through 3d Porous Media
The Apollonian packing represents the extreme case of a perfect filling of spherical beads with a power-law distribution of sizes. The space between disks is a "self-inverse" fractal of dimension $d_f$ (Mandelbrot, 1982).

Apollonius of Perga
(ca. 262 BC – ca. 190 BC)

Fractal Dimension - Two-dimensional Apollonian packing
Rigorous bounds (Boyd, 1973): $1.300\ 197 < d_f < 1.314\ 534$
Numerical (Manna & Herrmann, 1991): $d_f = 1.305\ 684$
Apollonian Packings are rotationally frustrated!

Apollonian

Generation 0
A toy model for turbulence?
Surface Quasi-Geostrophic Turbulence
Optimal Synchronizability of Bearings

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The Dynamics of Bearings

✓ Sliding friction is suppressed by ensuring that loops of touching disks have an even number of constituents. In this way, clockwise turning disks only touch counterclockwise rotating ones and vice-versa.

✓ At steady state, the tangential velocity is the same for all contacts.

✓ Consider a bearing of N rotors. The equation of motion for the angular velocity $\vec{\omega}_i$ of rotor i can be written as

$$I_i \dot{\vec{\omega}}_i = \sum_j r_i \vec{r}_{ij} \times \vec{F}_{ji}$$

$I_i$ - rotational inertia

$r_i$ - radius of rotor i

$\vec{F}_{ji}$ - force of rotor j on the surface of i

✓ $\vec{F}_{ji}$ is a dissipative force proportional to the relative velocity at the contact point

$$\vec{F}_{ji} = \sigma (\vec{v}_j - \vec{v}_i) = -\sigma (\vec{\omega}_j \times r_j \vec{r}_{ij} + \vec{\omega}_i \times r_i \vec{r}_{ji}), \text{ where } \vec{r}_{ji} = -\vec{r}_{ij}$$

✓ We introduce the constitutive relation between mass and radius,

$$m_i = 2ar_i^\alpha \quad \Rightarrow \quad I_i = ar_i^{\alpha+2}$$
It is possible to quantify how the energy dissipation rate is distributed among disks. The relative dissipation of disk $i$ in the eigenmode $k$ is given by,

$$Q_{i}^{k} = \frac{r_{i}^{\alpha} \vec{x}_{k}(i)^{2}}{\sum_{j} r_{j}^{\alpha} \vec{x}_{k}(j)^{2}}.$$

The image shows the energy dissipation for different values of $\alpha$. The scale on the bottom represents the dissipation rate, with $10^{-4}$ to $10^{-8}$ indicating the range of values. The diagrams illustrate the distribution of energy dissipation for $\alpha = 0$, $\alpha = 1$, and $\alpha = 2$. The color gradient corresponds to the dissipation rate, with orange indicating higher dissipation and blue indicating lower dissipation.
Turbulent flows are composed of a cascade of vortices of different sizes through which energy is passed down from larger to smaller scales until, at a sufficiently tiny length scale, the kinetic energy of the fluid is dissipated into heat by viscous friction.

\[ \text{slope} = -\frac{5}{3} \]

Lewis Richardson
For our numerical analysis, we consider a cubic box containing a non-Newtonian fluid and subjected to periodic boundary conditions in all three directions.

The mathematical formulation of the fluid mechanics is based on the assumptions that we have an incompressible fluid flowing under isothermal conditions.

**Continuity Equation**
\[ \nabla \cdot u = 0 \]

**Momentum Conservation Equation**
\[
\rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \nabla \cdot T + \Gamma
\]

\[ T = 2\mu(\dot{\gamma})\ E \quad \text{deviatoric stress tensor} \]
\[ E = \left( \nabla u + \nabla u^T \right) / 2 \quad \text{strain rate tensor} \]
\[ \dot{\gamma} = \sqrt{2E : E} \quad \text{second principal invariant} \]
Non-Newtonian Turbulence
Constitutive Relation - Rheology

cross power-law fluids

\[
\mu = \begin{cases} 
\mu_1, & \mu < \mu_1 \\
K\dot{\gamma}^{n-1}, & \mu_1 < \mu < \mu_2 \\
\mu_2, & \mu > \mu_2
\end{cases}
\]

\( n = 1 \rightarrow \text{Newtonian fluid} \)
\( n > 1 \rightarrow \text{shear-thickening} \)
\( n < 1 \rightarrow \text{shear-thinning} \)

✓ A central assumption explicitly involved in the theoretical construct of the Kolmogorov's K41 theory is that the fluid flow at a sufficiently large Reynolds number is in a homogeneous and locally isotropic state, the so-called fully developed turbulence, that can be described in terms of universal statistical properties.

\[
\Gamma = \rho \left( u - \langle u \rangle \right) / \tau
\]

linear forcing term

\( \tau \rightarrow \text{turnover time scale} \)

Non-Newtonian Turbulence
Direct Numerical Simulations (DNS)

✓ The numerical solution of for the time evolution of the local velocity and pressure fields is obtained through the open source DNS code Gerris, which is based on a second-order finite-volume scheme applied to an adaptively refined octree mesh.

✓ The maximal refinement level was set to eight subdivision steps, corresponding to a 256-cube discretization of our triple periodic box.

**Defining vortices with the $\lambda_2$-criterion**


✓ $\lambda_2$ is the second eigenvalue of the tensor $M = E^2 + Q^2$, where

$$Q = (\nabla u - \nabla u^T)/2$$

A vortex is defined as a connected region in space with at least two negative eigenvalues of $M$, thus leading to the criterion,

$$\lambda_2 < 0$$

✓ Vortices are then identified as clusters of cells in the numerical mesh for which $\lambda_2 \leq \lambda^*_2$, where $\lambda^*_2 \leq 0$ is a given threshold value. The smaller the prescribed parameter, the smaller is the average volume which encloses the vortex cores in the system.
Non-Newtonian Turbulence
Stationary State

\[ n = 1.5 \] (shear-thickening)

\[ \lambda_2^* = -10^{-5} \] (white contours)
The rheology has negligible impact on the statistical signature of the non-Newtonian turbulent flow. This agrees with the prediction that the structure of Newtonian turbulence at the inertial subrange is robust.

A non-Newtonian constitutive relation provides an additional degree of freedom which allows the system to dissipate energy in different ways.

\[ \epsilon_n = \frac{\mu (\dot{\gamma}) \dot{\gamma}^2}{\rho} \quad \text{viscous dissipation in a cell} \]

The dissipated energy is typically smaller inside the vortices than outside them, regardless of rheology.
Non-Newtonian fluids undergoing fully developed turbulence self-organize in distinctive dissipative regimes at the microscopic level so as to display vortex distributions that are statistically identical to that of Newtonian turbulence.

\( \phi_n = \left( \frac{\int_{\lambda_2^*}^{\lambda_2^{max}} \varepsilon(\lambda_2) d\lambda_2}{\int_{\lambda_2^{min}}^{\lambda_2^*} \varepsilon(\lambda_2) d\lambda_2} \right) \)

The ratio between the total energies dissipated outside and inside the vortices, calculated over the entire box and many snapshots.

Self-organization !!!
Newtonian Turbulence

Anomalous Scaling

\[ S_m^*(r) = \langle [u(x + r) - u(x)] \cdot \frac{r}{r} \rangle^m \]

structure function of order \( m \)

Andrei Kolmogorov

The Kolmogorov 4/5 Law

average rate of energy dissipation per unit mass

The K41 Theory

Chen et al., J. Fluid Mech. 533, 183 (2005)
Non-Newtonian Turbulence

Deviations from K41 Theory

$(\xi_m - m/3)/(m/3)$

$m$

Nonuniversal Anomalous Scaling !!!
Thank you!